Projection, entanglement and nonlocality of photon-number entangled states generated in Kerr media.

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Motivated by the proposed alternative nonlocality quantifier [1], and a wide family of symmetric states ICPS [2], we preset grasp application of those tools in the analysis of a real physical system. We analyze photon-number entangled states which are generated in Kerr media with optical parametric pumping [3]. We combine nonlocality measures and ICPS states. First, we project the density matrix of our physical system onto the family of ICPS by averaging over symmetries (*twirling*). Next, for each time step of the considered time evolution of our system, we calculate the negativity and negativity of projected states to determine the lower bound of entanglement and compare it with the actual one. Finally, we compare those results with the lower bound of nonlocality calculated with the volume of violation.

The model

We consider a system composed of two nonlinear Kerr oscillators mutually coupled by parametric pumping. In a given system, pairs of photons via spontaneous parametric down-conversion are produced. The Hamiltonian describing the analyzed system takes the following form [3]:

$$\hat{H}_{int} = \frac{\chi_a}{2}(\hat{a}^\dagger)^2\hat{a}^2 + \frac{\chi_b}{2}(\hat{b}^\dagger)^2\hat{b}^2 + g\hat{a}^\dagger\hat{b}^\dagger + \overline{g}\hat{a}\hat{b} + G\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b} \quad (1)$$

where the first two terms of the Hamiltonian describe the nonlinear Kerr-type oscillators characterized by the nonlinearity constants χ_a and χ_b. The last term is the Kerr cross term, whereas the other two are related to the two-mode parametric process. Parameter g represents the strength of the external field. Operators $\hat{a}^\dagger(\hat{b}^\dagger)$ and $\hat{a}(\hat{b})$ are the creation and annihilation operators (subsystems $A(B)$).

The considered system is initially in the vacuum state and $g < \chi_a + \chi_b$, hence the time evolution is limited to three possible states. The truncated wave function can be expressed as:

$$|\psi(t)\rangle_{cut} = c_{00}(t)|00\rangle + c_{11}(t)|11\rangle + c_{22}(t)|22\rangle, \quad (2)$$

The trunctuation accuracy is measured by the function $F(t)$, where:

$$F(t) = |c_{00}|^2 + |c_{11}|^2 + |c_{22}|^2. \quad (3)$$

From Fig. 1, we can see that the deviation of the function $F(t)$ from unity is of the order of 10^{-5}, and hence, the states with a higher number can be neglected.

The projection

The family of ICPS states [2] is characterized by six real numbers (five independent). We projected the density matrix $\rho(t)$ corresponding to the system described by Eq.1) onto the family of ICPS states $\tilde{\rho}(t)$ ($\mathbb{P} : \rho(t) \to \tilde{\rho}(t)$). The operation of projection (often termed *twirling*) [4] cannot increase the degree of entanglement [5]. Let $\rho(t) = |\psi(t)\rangle_{cut} \otimes_{cut} \langle\psi(t)|$. Then the only non-zero elements of projection density matrix $\tilde{\rho}(t)$ are given by: $a_1 = \frac{|c_{00}|^2+|c_{11}|^2}{2}$, $b_1 = Re(\overline{c_{00}c_{11}})$, $b_2 = Re(\overline{c_{00}c_{22}}) + Re(\overline{c_{11}c_{22}})$ and $a_4 = 1 - 2a_1$. The time evolutions of these parameters are presented in Fig. 2. We can see that the two most fluctuating in time parameters are b_1 and b_2, which are related to off-diagonal elements of the density matrix $\tilde{\rho}(t)$.

The entanglement and volume of violation

To quantify entanglement generated in the analyzed system, we use the negativity [6]:

$$N(\rho) = \frac{\| \rho^{T_A} \|_1 - 1}{2}, \quad (4)$$

where $\| \cdots \|_1$ means the trace norm of the matrix, and ρ^{T_A} is a partially transposed density matrix. Negativity for projections $\tilde{\rho}(t)$ can be expressed as:

$$N(\tilde{\rho}(t)) = |b_1| + 2|b_2| \quad (5)$$

The time evolution of both negativities, that is, $N(\rho(t))$ and $N(\tilde{\rho}(t))$, is illustrated in Fig. 3. The analyzed period is large enough to observe a periodical behavior in the amount of entanglement generation. Initially starting from zero, the negativity $N(\rho(t))$ quite fast breaks the limit of 0.3 and converges to its maximum around 0.55.

In our analysis, we consider bipartite quantum systems $d \otimes d$. For such a system, we use an alternative measure of nonlocality proposed by Fonseca and Parisio [1]:

$$V(\rho, I) = \int_{\Gamma_{\rho,I}} d^n x, \quad (6)$$

where $d^n x = dx_1 \ldots dx_n$, I is an arbitrary Bell-type inequality, X is a space of all possible configurations of parameters x_i, and subspace $\Gamma_{\rho,I} \subset X$ contains all possible configurations x_i leading to violation of inequality I. Here we use CGLMP inequality [7] and the set of local observables M_1 (to see more details please check [8]). The evolution of V is more polarized in the sense, that there is a longer time where there is no nonlocality detected. This is contrary to entanglement which, although often small, is detected almost all time.

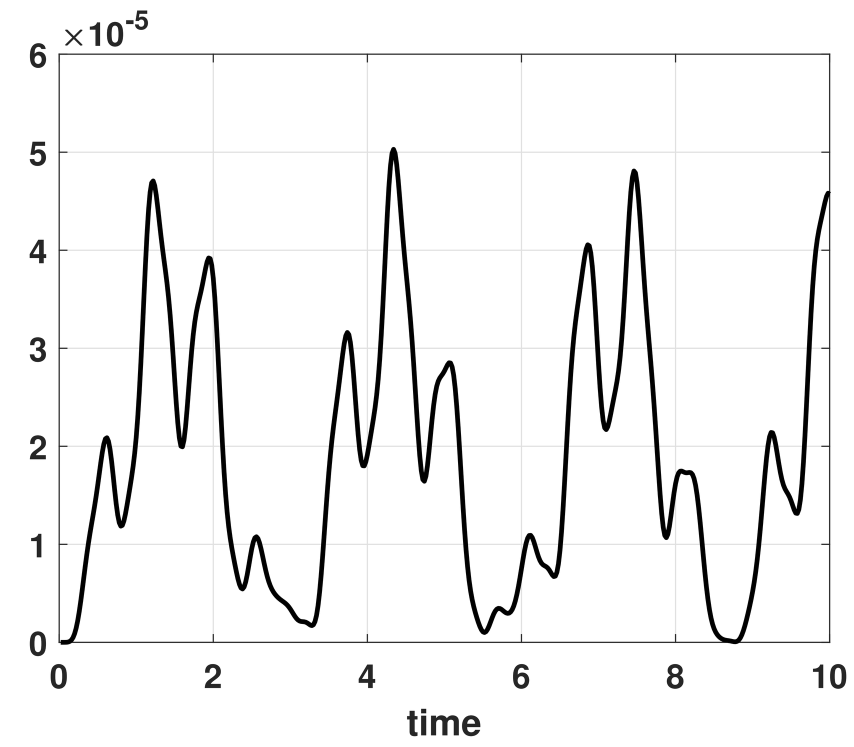

Figure 1: The time-evolution of the $1 - F(t)$ function for $|\psi(t = 0)\rangle = |0\rangle_a|0\rangle_b$, $\chi_a = \chi_b = 1$, $g = 0.6$, and $G = 2\chi_a$. Time is scaled in the units of $1/\chi_{a,b}$.

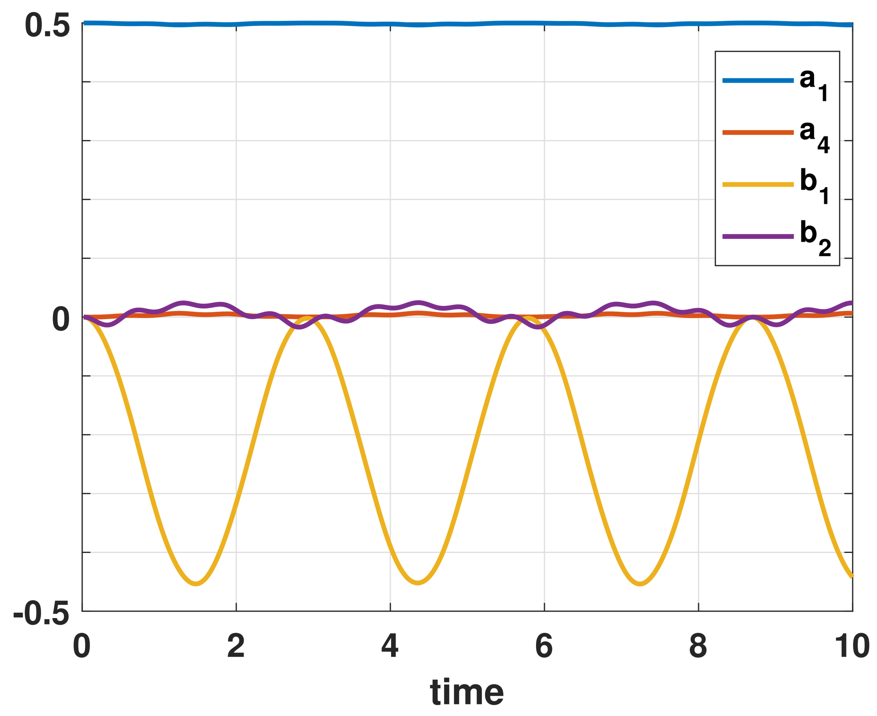

Figure 2: The time-evolution of the a_1, a_4, b_1, b_2 as elements of projected density matrix $\tilde{\rho}(t)$. The values of parameters are the same as for previous figure.

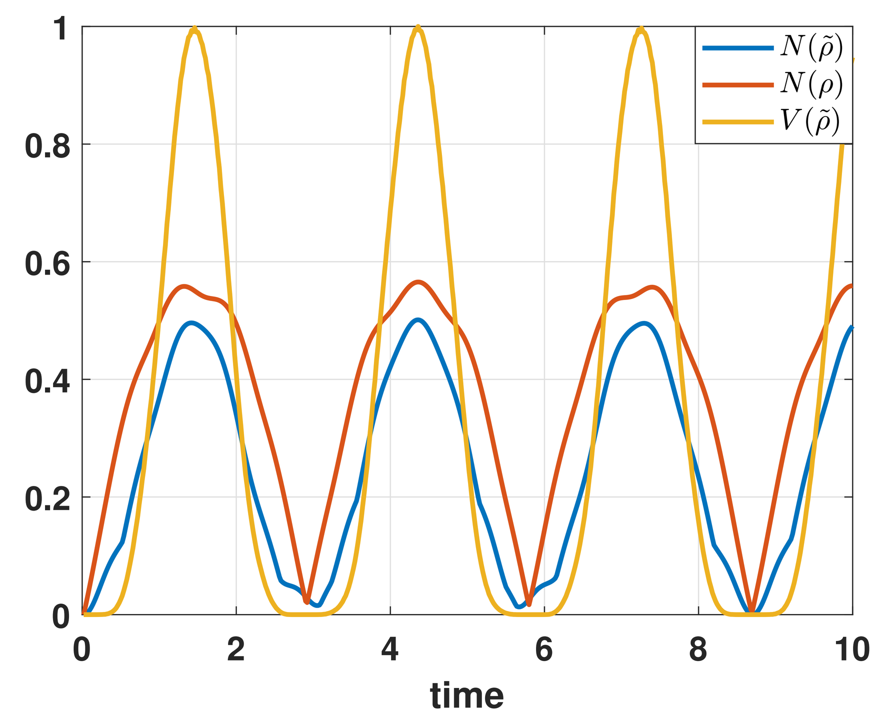

Figure 3: Time evolution of $\mathcal{N}(\rho), \mathcal{N}(\tilde{\rho})$ and $V(\rho)$. Volume of violation was normalized. The values of parameters are the same as for previous figures.

Summary

To the given system (1) we have applied projection to symmetric ICPS states, for which, we quantified both entanglement and volume of violation. They serve as a lower bound of quantum correlations of the actual system for which we calculated entanglement as well.

- For our model, we can see that $N(\tilde{\rho}(t))$ follows quite accurately the periodical change of $N(\rho(t))$.
- The mean value of the difference $N(\rho(t)) - N(\tilde{\rho}(t))$ is 0.1218, which is around 25% of the mean value of $N(\rho(t))$.
- For the normalized time evolution of V, we can observe the same periodicity as in the case of $N(\rho(t))$, and $N(\tilde{\rho}(t))$, however at low values of negativity we do not observe any nonlocality.

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