The heat conductivity equation:

$$
\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T(x_i, t)}{\partial x_j} \right) + \dot{Q}
$$

where the volumetric efficiency of the internal heat source is follows:

$$
\dot{Q} = \rho_S L \frac{df_S(T(x_i, t))}{dt}.
$$

Finally, the heat conductivity equation is in a without-source form:

$$
\rho C_{ef} \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T}{\partial x_j} \right)
$$

Additionally, assuming a linear function of the solid phase fraction  $(f_s)$ , we obtain the following *Ce*<sup>f</sup> form for the mushy zone:

The effective specific heat (Cef) for each phase is determined as follows:

$$
C_{ef}(T) = \begin{cases} c_L(T), & T > T_L, \\ c_{LS}(T) - L\frac{\partial f_s}{\partial T}, & T_s < T < T_L, \\ c_S(T), & T < T_s. \end{cases}
$$

$$
f_s = \frac{T_L - T}{T_L - T_S}
$$
\n
$$
C_{ef} = c_{LS} + \frac{L}{T_L - T_S}
$$

where: *T* is the temperature [K], *t* is the time [s],  $\rho(T)$  is the density [kg/m<sup>3</sup>],  $\lambda(T)$  is the thermal conductivity coefficient [W/(m·K)],  $T_L, T_S$  are the temperature of liquids and solids line, respectively [K],  $c<sub>L</sub>$ ,  $c<sub>L</sub>$ ,  $c<sub>S</sub>$  are the specific heat of liquid phase, mushy zone and solid phase, respectively  $[J/(kg·K)]$ , *L* is the latent heat of solidification  $[J/kg]$ ,  $x_i$  are the coordinates of a node position [m],  $T_{in}$  - the initial temperature [K],  $T_a$  - the ambient temperature [K],  $\alpha_M$  -

the heat transfer coefficient between the ambient and the mould  $[W/(m^2K)]$ ,  $\lambda_M$  - the thermal conductivity coefficient of mould  $[W/(m \cdot K)]$ , *n* - the outward unit normal surface vector.

Parameters of the casting process



### Results of numerical simulations



*Leszek Sowa*

Fig.4.Temperature field above the solidus temperature at: a) *t*=325 s, I variant, b) *t*=375 s, II variant

# Mathematical model

# **Numerical modeling of basic physical phenomena during the creation process of a casting-riser system**

#### Final remarks and conclusions

 $(x_i, t_0) = T_0(x, y, z) = \left\{ T_{in} \text{ in } \Omega_L. \right.$  $\left|T_M\right|$  in on  $\overline{\phantom{a}}$  $\left(T_M$  *in*  $\Omega_M$  $\Big\}$  $\overline{\phantom{a}}$  $T(x_i, t_0) = T_0(x, y, z) = \left\{ T_{in} \text{ in } \Omega_L \right\}$  $\left[T_M$  on  $\Gamma_G\right]$ The initial conditions for temperature fields

- This paper presents the mathematical model and the numerical simulations results of solidification of the three-dimensional castingriser-mold system. The influence of the riser shape on the location of the solidification end of the casting-riser system was assessed. Numerical calculations were performed for two shapes of the riser: cylindrical (I variant ) and conical (II variant), estimating their suitability for feeding the shrinking casting during the solidification process.
- It was observed that in the final stage of the casting-riser solidification process, a solidus line closed in the upper part of the casting is visible in the case of a cylindrical riser. This suggests the formation of internal defects at this place in the form of a shrinkage cavity (Figs. 3 and 4a). Such a situation was not observed in the case of using a cone-shaped riser (Fig. 4b). In this case, the end of solidification took place in the riser which is desirable as the riser can be cut off and reused. It also proves that the inverted cone-shaped riser fulfilled its task and the casting was made without internal defects.



Applications of Physic in Mechanical and Material Engineering APMME 2021 Częstochowa, 19 luty 2021 rok



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Material properties of the casting - cast steel







#### a) b) Temp (Kelvin) Temp (Kelvin) 1805 1806 1692 1692 1578 1579 1465 1466 1351 1352 1238 1239 1124 1125 1011 1012 897 899 784 785 670 672 557 559 443 445 330 332

Fig.2.Temperature distribution at *t*=200 s, I variant





Fig.3. Temperature distribution at *t*=325 s, I variant

## Numerical calculations

Fig.1. The cross-section of the casting-mould system and identification of sub-regions of the considered region

Material properties used in the calculations for other regions



$$
\frac{\partial T}{\partial n}|_{\Gamma_{1-1}} = 0, \qquad \lambda_M \frac{\partial T_M}{\partial n}|_{\Gamma_M} = -\alpha_M \Big( T_M |_{\Gamma_M} - T_a \Big),
$$

$$
\lambda_S \frac{\partial T_S}{\partial n}|_{\Gamma_{G-}} = \lambda_G \frac{\partial T_G}{\partial n}|_{\Gamma_{G-}}, \qquad \lambda_G \frac{\partial T_G}{\partial n}|_{\Gamma_{G+}} = \lambda_M \frac{\partial T_M}{\partial n}|_{\Gamma_{G+}},
$$

The boundary conditions