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## APPLICATIONS OF PHYSICS IN MECHANICAL AND MATERIAL ENGINEERING

# NATURAL FREQUENCY OF AN ELASTICALLY MOUNTED COLUMN AXIALLY LOADED WITH A MASS ELEMENT

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*In this work, the influence of the loading method of an elastically mounted column on its natural frequency was investigated. The classic way to load these types of systems is to apply an axial force. In the presented approach, the load of the system in the form of a mass element was adopted, which much better reflects the real slender support system, whose task is to support a structure with a specific own weight. During the formulation of the boundary problem, Hamilton principle and perturbation small parameter method were used. A series of numerical simulations were carried out, taking into account the influence of the system parameters and the method of loading on the non-linear natural frequency. The main task was to determine the impact of the change in stiffness at mounting points of the system ends on the dynamic behaviour of the structure. It was shown that these stiffness have a significant impact on the natural frequency. It was also indicated that in the problem formulated in this way, the amplitude level of the induced system vibrations is of significant importance - which is not taken into account in the case of a force load. The knowledge of potential resonance frequencies in the case of slender support systems is one of the basic data taken into account in the design process of this type of structures due to their susceptibility to vibrations.*

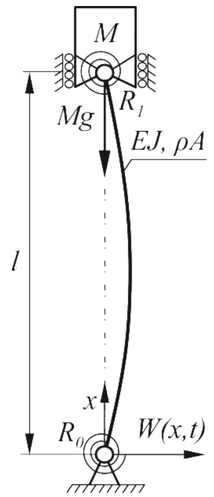


*Fig. 1. Scheme of the considered system*

The column elastically mounted on both sides and loaded by the mass element is under consideration. The load fulfils the Euler load conditions with additionally taken into account the longitudinal inertia of the loading element. The elastic mounting of the system is modelled by means of two rotational springs.

*W(x,t)* – displacement in the transverse direction
*l* – length of the system
*M* – mass element
*Mg* – load of the system due to the action of element *M*
*EJ – flexural rigidity*
*ρA* – unit mass
*$R_0$, $R_1$* – rotational springs

Kinetic *T* and potential *V* energy:

$$V=\frac{1}{2}\int_0^l EJ\left(\frac{\partial^2 W(x,t)}{\partial x^2}\right)^2 dx + MgU(l,t) +$$
$$+\frac{1}{2}\int_0^l EA\left(\frac{\partial U(x,t)}{\partial x}+\frac{1}{2}\left(\frac{\partial W(x,t)}{\partial x}\right)^2\right)^2 dx +$$
$$+\frac{1}{2}R_0\left(\left.\frac{\partial W(x,t)}{\partial x}\right|_{x=0}\right)^2+\frac{1}{2}R_1\left(\left.\frac{\partial W(x,t)}{\partial x}\right|_{x=1}\right)^2$$
$$T=\frac{1}{2}\rho A\int_0^l\left(\frac{\partial W(x,t)}{\partial t}\right)^2 dx+\frac{1}{2}M\left(\left.\frac{\partial U(x,t)}{\partial t}\right|^{x=l}\right)^2$$

Dimensionless parameters:

$$\xi=\frac{x}{l},\; w(\xi,\tau)=\frac{W(x,t)}{l},$$
$$u(\xi,\tau)=\frac{U(x,t)}{l},\; \tau=\omega t,$$
$$\theta=\frac{Al^2}{J},\; k^2(\tau)=\frac{S(t)l^2}{EJ},$$
$$\Omega^2=\frac{\omega^2(\rho A)l^4}{EJ},$$

Formulation of the problem is based on the Hamilton principle. After appropriate mathematical transformations and taking into account the dimensionless parameters, following boundary conditions and equations of motion can be obtained:

$$\frac{k^2(\tau)EJ}{l^2}-M\omega^2 l\frac{\partial^2 u(\xi,\tau)}{\partial\tau^2}=0 \qquad w(0,\tau)=w(1,\tau)=u(0,\tau)=0$$

$$\frac{EJ}{l}\left.\frac{\partial^2 w(\xi,\tau)}{\partial\xi^2}\right|^{\xi=1}+R_1\left.\frac{\partial w(\xi,\tau)}{\partial\xi}\right|^{\xi=1}=0 \qquad \frac{\partial^4 w(\xi,\tau)}{\partial\xi^4}+k^2(\tau)\frac{\partial^2 w(\xi,\tau)}{\partial\xi^2}+\Omega^2\frac{\partial^2 w(\xi,\tau)}{\partial\tau^2}=0$$

$$\frac{EJ}{l}\left.\frac{\partial^2 w(\xi,\tau)}{\partial\xi^2}\right|_{\xi=0}-R_0\left.\frac{\partial w(\xi,\tau)}{\partial\xi}\right|_{\xi=0}=0 \qquad \frac{\partial}{\partial\xi}\left(\frac{\partial u(\xi,\tau)}{\partial\xi}+\frac{1}{2}\left(\frac{\partial w(\xi,\tau)}{\partial\xi}\right)^2\right)=0$$

The non-linear term appearing in equation of motion in longitudinal direction is developed into a series of small vibration amplitude parameter *ε*. Then the equations are grouped with respect to the same powers of the small parameter. The obtained equations are solved sequentially and based on them following parameters are determined:

- linear component of internal force in the column,
- linear component of the natural frequency,
- non-linear component of internal force in the column,
- non-linear component of natural frequency.

The results of the numerical calculations were presented with the use of dimensionless parameters:

$$\lambda=\frac{M}{M_E},\;\Omega^*=\sqrt{\Omega_0^2+\varepsilon^2\Omega_2^2},\;\zeta_A=\frac{Amp}{r},\;r_0=\frac{R_0 l}{EJ},\;r_1=\frac{R_1 l}{EJ},$$

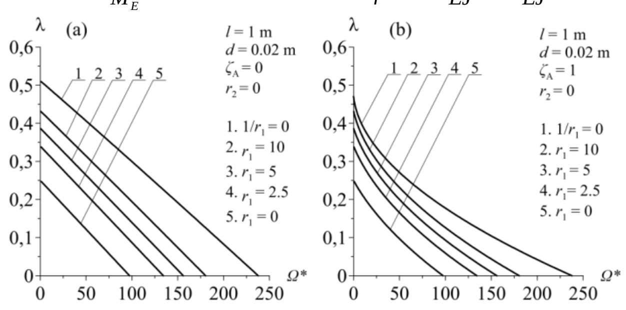


*Fig.2. The influence of one-side mounting rigidity on natural frequency: a) $\zeta_A$ = 0, b) $\zeta_A$ = 1*

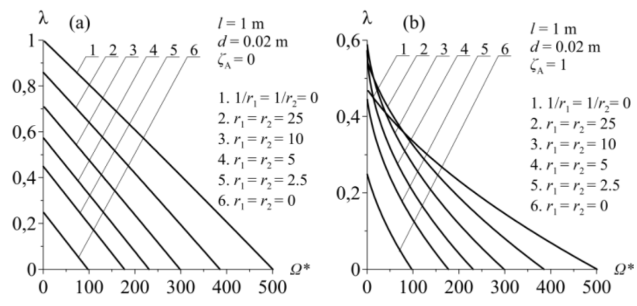


*Fig.3. The influence of two-sides mounting rigidity on natural frequency: a) $\zeta_A$ = 0, b) $\zeta_A$ = 1*

On the basis of the obtained results, it was found that an increase in the stiffness in the supports causes the characteristic curves to shift towards higher values. This is because the overall system stiffness has increased. The course of characteristic curves in the linear problem is linear. Increasing the stiffness in the supports may increase the critical load of the system (*λ* for Ω*=0). The increase in stiffness when considering the same column load causes an increase in the natural frequency. Controlling the stiffness of the support can be one way to actively counteract resonance.
Taking into account the non-linear problem (the amplitude effect) changes the course of the characteristic curves from linear to non-linear. It can be observed that with certain values of stiffness the critical load of the system decreases. This is mainly due to the vibration amplitude and increased system stiffness. As shown in [10], an excessive increase in amplitude may result in a reduction of the critical load. Moreover, the influence of the set amplitude level on the system with higher stiffness is greater, therefore, with higher stiffness in the supports, a reduction of the critical load with regard to the linear problem can be observed. In an extreme case, the curves may intersect (cf. Fig. 3b)). In this case, the increase in stiffness in relation to the same level of vibration amplitude resulted in a significant reduction of the critical load.

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