Comparison of results from an in-house solidification convection model with a standard benchmark

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Introduction

Author presents the performance of a developed model against standard benchmark of solidification commonly found in literature. Developed model is based on Navier-Stokes equation and energy equation with convection term and also takes into account latent heat of solidification. This set of equations is numerically solved by the Finite Element Method. In order to overcome numerical difficulties arising from solving Navier-Stokes equation, Streamline Upwind Petrov Galerkin and Pressure Stabilized Petrov Galerkin type of FEM formulation is used. Resulting numerical model is implemented in C++ programming language with the use state of the art numerical libraries which allows it to be run on High Performance Computers.

Mathematical formulation

Energy transfer:

$$\boldsymbol{c}^{*}\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{t}} + \rho \boldsymbol{c}\left(\boldsymbol{u}\cdot\nabla\right)\boldsymbol{T} = \lambda\nabla^{2}\boldsymbol{T}$$

Mass transfer:

$$\rho\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}} + \rho\left(\left(\boldsymbol{u}\cdot\nabla\right)\boldsymbol{u}\right) - \nabla \boldsymbol{p} + \rho\mu\left(\left(\nabla \boldsymbol{u}\right) + \left(\nabla \boldsymbol{u}\right)^{T}\right) + \rho\mu\frac{\boldsymbol{f_l}}{\boldsymbol{K_\epsilon}}\boldsymbol{u} = \rho \boldsymbol{f}$$

$$\nabla \boldsymbol{u} = \boldsymbol{0}$$

where:

$$\boldsymbol{c}^{*} = \frac{\boldsymbol{H}^{n} - \boldsymbol{H}^{n-1}}{\boldsymbol{T}^{n} - \boldsymbol{T}^{n-1}},$$

$$\boldsymbol{f} = -\beta \boldsymbol{g}\left(\boldsymbol{T} - \boldsymbol{T_0}\right),$$

$\boldsymbol{T}$ is temperature, $\boldsymbol{u}$ is velocity vector, $\boldsymbol{p}$ is pressure, $\boldsymbol{H}$ is enthalpy, ρ is density, $\boldsymbol{c}$ is specific heat, λ is thermal conductivity coefficient, μ is dynamic viscosity, β is expansion coefficient, $\boldsymbol{g}$ is gravitational acceleration, $\boldsymbol{T_0}$ is reference temperature, $\boldsymbol{K_\epsilon}$ is the permeability of the mushy zone and $\boldsymbol{f_l}$ is liquid fraction

Problem setup

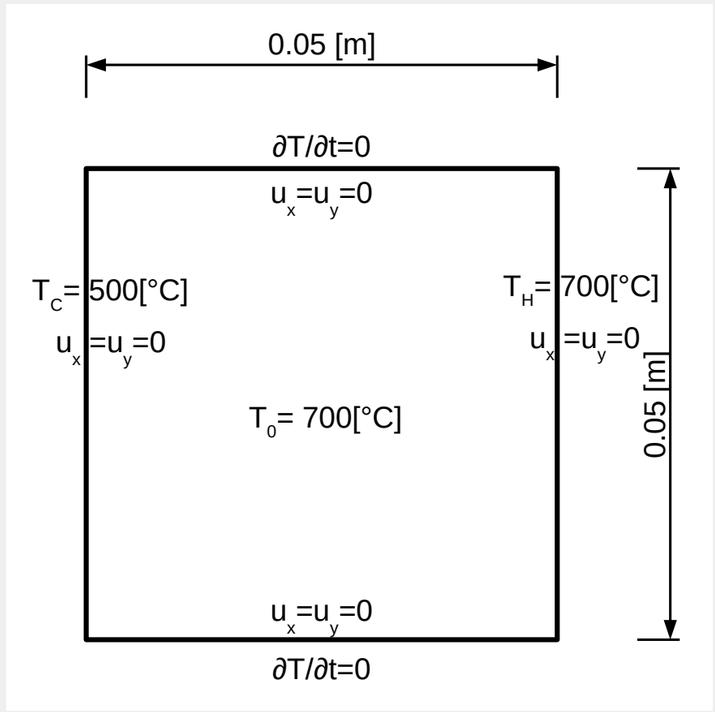

Figure: Boundary conditions for the benchmark problem

Thermophysical properties

Quantity	Value	Unit
Conductivity	100	W/(m °C)
Specific heat	1000	J/(kg °C)
Density	2500	kg/m^3
Latent Heat	400000	J/kg
Viscosity	0.0025	kg/(m s)
Coefficient of thermal expansion	$4.0 \cdot 10^{-5}$	kg/m^3
Solidus temperature	550	°C
Liquidus temperature	650	°C
Melting point of pure aluminium	675	°C
Partition coefficient	0.14	—

Temperature fields

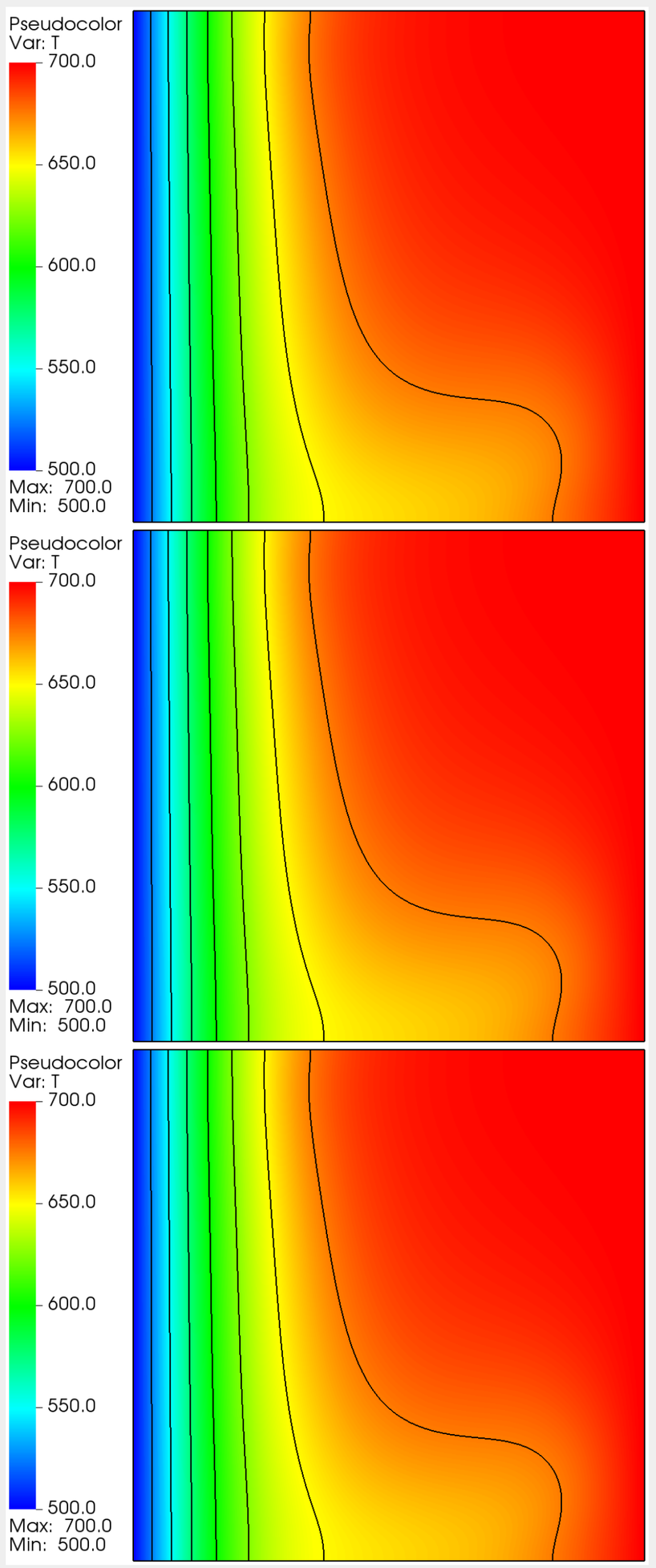

Figure: Temperature field after (from top to bottom) 5 [s], 10 [s], and 15 [s]

Remarks

Based on temperature profiles, it can be seen that qualitative behaviour of presented model is correct. It can be observed that layer of solidified metal is thicker at the bottom of region. This behavior is in agreement with physical observations. Moreover, obtained temperature profiles are also in good agreement with temperature profiles presented by the other authors[1].

1. C.R. Swaminathan, V.R. Voller, *Int. J. Num. Meth. Heat Fluid Flow,* ***3****, 3 (1993), DOI:10.1108/eb017528*