Partial Energy Transfer Model of Lamb Waves Scattering in Materially Isotropic Waveguides

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Abstract

The scattering phenomena of fundamental antisymmetric Lamb wave mode with a horizontal notch enabling the partial energy transfer (PET) option is being solved within the framework of this paper. The PET functionality for a given waveguide is realized using the material interface. The energy scattering coefficients are identified with use of two different methods, namely a hybrid approach, utilizing the finite element method (FEM) and the general orthogonality relation, and the semi-analytical approach, which combines the modal expansion technique with the orthogonal property of Lamb waves.

Mode matching technique

The total stress/displacement field resulting from interaction between Lamb wave modes and a given geometry can be expressed as a sum of individual stress/displacement components, which are weighted by complex amplitudes α_n^+ / α_n^- for n-th Lamb wave mode with wavenumber k^n:

$$\begin{bmatrix} u(x,z) \\ w(x,z) \\ \sigma_{xx}(x,z) \\ \sigma_{zx}(x,z) \end{bmatrix} = \sum_{n\in\mathbb{N}} \alpha_n^+(x) \begin{bmatrix} \tilde{u}^{n+}(z) \\ \tilde{w}^{n+}(z) \\ \tilde{\sigma}_{xx}^{n+}(z) \\ \tilde{\sigma}_{zx}^{n+}(z) \end{bmatrix} + \sum_{n\in\mathbb{N}} \alpha_n^-(x) \begin{bmatrix} \tilde{u}^{n-}(z) \\ \tilde{w}^{n-}(z) \\ \tilde{\sigma}_{xx}^{n-}(z) \\ \tilde{\sigma}_{zx}^{n-}(z) \end{bmatrix}$$

where subscript $^-$ refers to left-going waves while subscript $^+$ refers to right-going waves. Using biorthogonality relation

$$\int_{t_h} -\tilde{u}^n(z)\tilde{\sigma}_{xx}^m(z) + \tilde{w}^m(z)\,\tilde{\sigma}_{zx}^n(z)\mathrm{d}z = \Lambda_n \delta_{mn},$$

it is possible to explicitly derive the global scattering matrix, which contains the information related to the complex amplitudes of left/right-going wave modes with respect to considered waveguide sections.

Verification of PET model

- Horizontal notch in 10 mm thick aluminum plate (ρ=2660 kg/m^3, E=68.5 GPa, ν=0.34).
- L=0.01 m, waveguide thick.: 2-5: 4mm; 3-6: 3mm; 4-7: 3mm. PET model applied on 3-6 waveguide using variable value of Young´s modulus to reach desired ratio of reflected/transmitted energy in this section.
- Incident mode/freguency: A0/150 kHz
- The problem has been num. solved using COMSOL software in frequency domain.

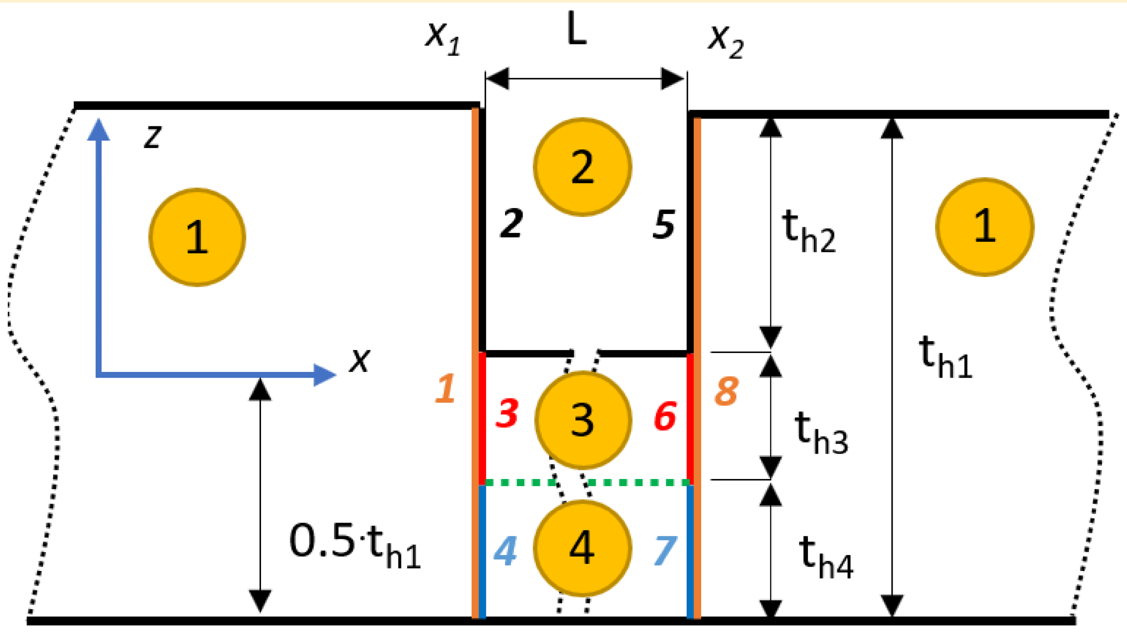

Fig. 1: ***Plate with horizontal notch***

Results

- The PET approach shows very good, if not excellent, agreement compared to its FEM counterpart, which served as reference.
- The maximum difference across the individual energy coefficients was not higher than 3% for the given range of reflected energy in sub-waveguide 3, i.e. 0-100%.
- The overall convergence character of the semi-analytical method incorporating the PET approach was among other influenced by the utilized value of the reflected energy in sub-waveguide 3.
- Lower values of the reflected energy, (up to 50%) induced rapid convergence, where the proper energy balance within the accuracy of 0.5% has been achieved even including only real wavenumbers for all considered waveguides.
- With increasing portion of reflected energy in sub-waveguide 3 tends the computational process be less effective leading to an unstable character for values, which are close to 100% of the reflected energy portion in given sub-waveguide. Such instability can be solved by utilizing reflection matrix as in sub-waveguide 4.
- Overall, the exploited semi-analytical approach, which incorporates the PET functionality, has proved to effectively simulate the driven partial energy transfer through selected waveguide section with the potential for the use in more complex applications.

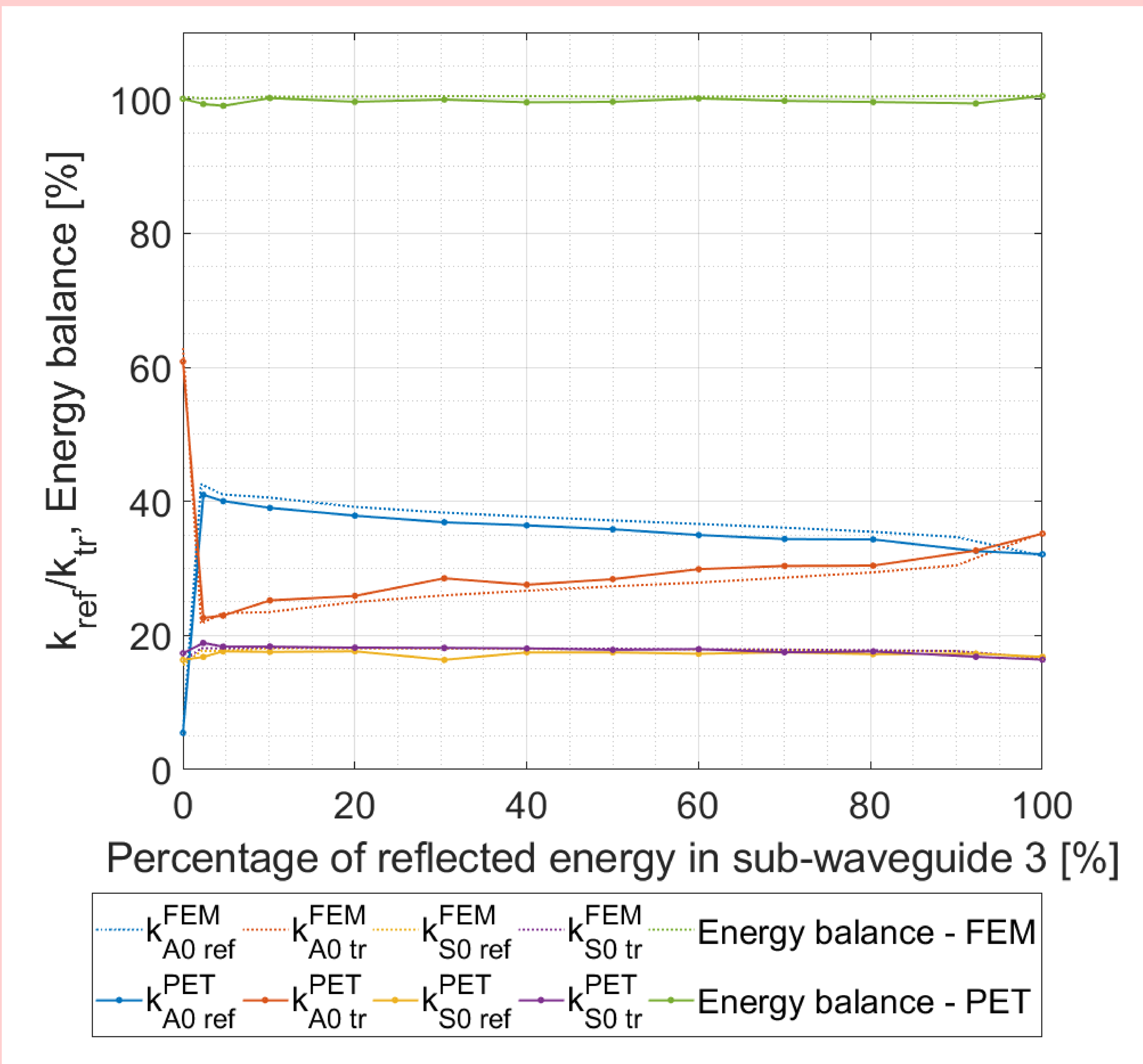

Fig. 2: ***Energy scattering coefficients***