APPLICATIONS OF PHYSICS IN MECHANICAL AND MATERIAL ENGINEERING APPME 2021

POLITECHNIKA CZĘSTOCHOWSKA
CZESTOCHOWA UNIVERSITY OF TECHNOLOGY

WYDZIAŁ INŻYNIERII MECHANICZNEJ I INFORMATYKI
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KMiPKM

FREE VIBRATIONS AND STABILITY OF A GEOMETRICALLY NONLINEAR STEEL COLUMN WITH A NON-PRISMATIC ELEMENT – THEORETICAL, NUMERICAL AND EXPERIMENTAL STUDIES

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PHYSICAL MODEL

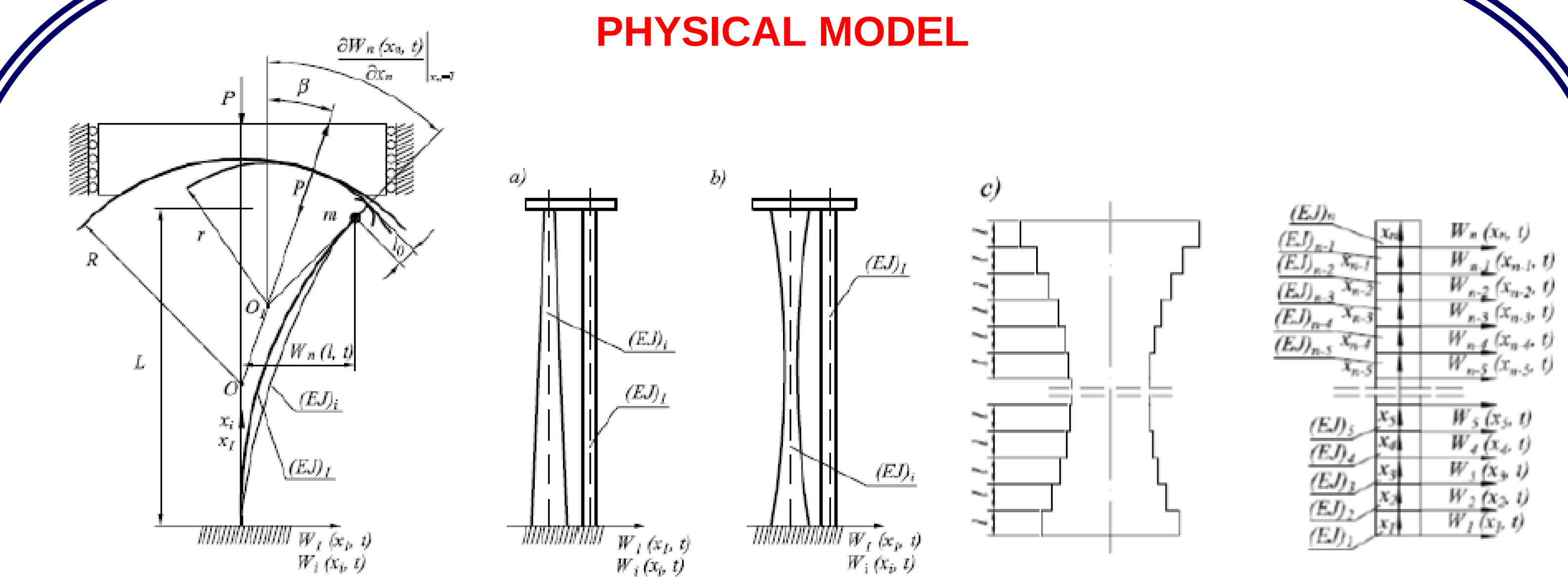

Fig.1. Scheme of **geometrically nonlinear column** with with nonprismatic rod approximated by a) linear function NN_1 b) polynomial of degree 2 NN_2; c) method of modeling nonprismatic rod.

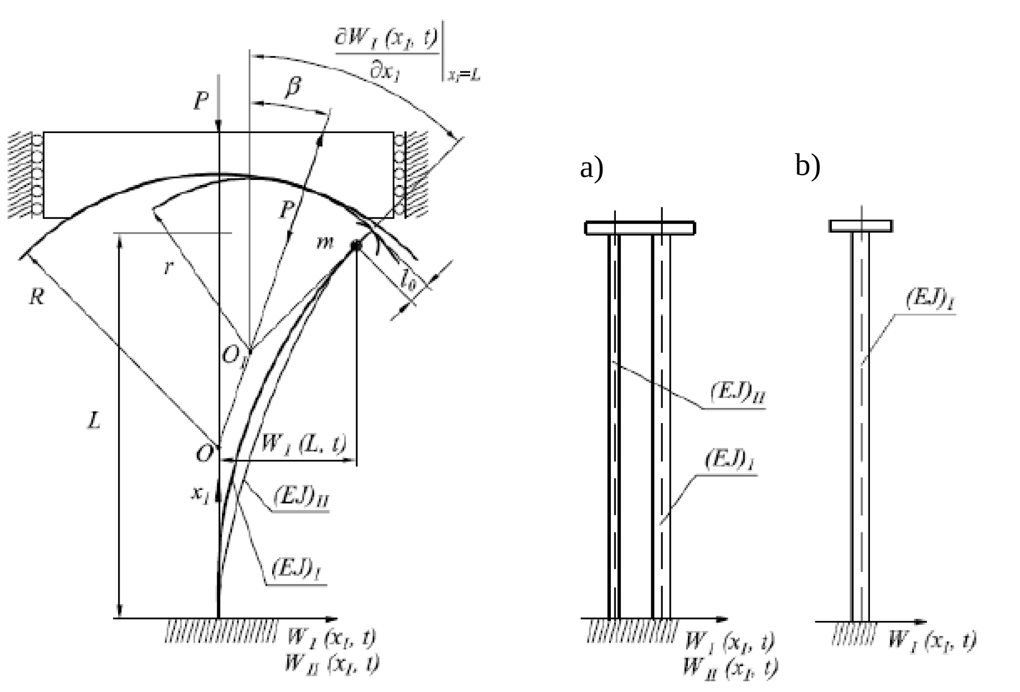

Fig.2. Schemes of comparative geometrically nonlinear column N

Approximating functions:

$$b(x) = 2a(Z)\cdot x + d,$$

$$b(x) = 2\left(a(p,q)\cdot[x-p]^2 + q\right)$$

$$\kappa = \frac{b_{pr}}{h}$$

Assumptions:

$$\sum_{k=I}^{II}(EJ)_k = const., \quad k = I, II$$

$$\mu = \frac{(EJ)_{II}}{(EJ)_I}$$

MATHEMATICAL MODEL

Mechanical Energy of the geometrically nonlinear column with non-prismatic rod under the generalized load with a force directed towards the positive pole:

$$V_1 = \frac{1}{2}(EJ)_I\int_0^L\left[\frac{\partial^2 W_I(x_I,t)}{\partial x_I^2}\right]^2 dx_I + \frac{1}{2}\sum_{i=1}^{n}(EJ)_i\int_0^l\left[\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}\right]^2 dx_i$$

$$V_2 = \frac{1}{2}(EA)_I\int_0^L\left[\frac{\partial U_I(x_I,t)}{\partial x_I} + \frac{1}{2}\left(\frac{\partial W_I(x_I,t)}{\partial x_I}\right)^2\right]^2 dx_I +$$

$$+\frac{1}{2}\left(\sum_{i=1}^{n}(EA)_i\int_0^l\left[\frac{\partial U_i(x_i,t)}{\partial x_i} + \frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2\right]^2 dx_i\right) + PU_I(L,t)$$

$$V_3 = \frac{1}{2}P\beta W_n(l,t)$$

$$V_4 = \frac{1}{2}P\left[\left.\frac{\partial W_n(x_n,t)}{x_n}\right|_{x_n=l} - \beta\right](r-l_0)\left.\frac{\partial W_n(x_n,t)}{x_n}\right|_{x_n=l}$$

$$T_1 = \frac{1}{2}(\rho A)_I\int_0^L\left[\frac{\partial W_I(x_I,t)}{\partial t}\right]^2 dx_I + \frac{1}{2}\sum_{i=1}^{n}(\rho A)_i\int_0^l\left[\frac{\partial W_i(x_i,t)}{\partial t}\right]^2 dx_i$$

$$T_2 = \frac{1}{2}m\left[\frac{\partial W_n(l,t)}{\partial t}\right]^2$$

Boundary problem was formulated on the basis of Hamiltons principle: $\int_{t_1}^{t_2}(\delta T - \delta V)\,dt = 0.$

The differential equations of motion in the axial and lateral direction were obtained in the following form:

$$(EJ)_i\frac{\partial^4 W_i(x_i,t)}{\partial x_i^4} + S_{II}(t)\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2} + (\rho A)_i\frac{\partial^2 W_i(x_i,t)}{\partial t^2} = 0, \quad i = 1, 2, \ldots, n,$$

$$(EJ)_I\frac{\partial^4 W_I(x_I,t)}{\partial x_I^4} + S_I(t)\frac{\partial^2 W_I(x_I,t)}{\partial x_I^2} + (\rho A)_I\frac{\partial^2 W_I(x_I,t)}{\partial t^2} = 0.$$

$$U_i(x_i,t) - U_i(0,t) = -\frac{S_{II}(t)}{(EA)_i}x_i - \frac{1}{2}\int_0^{x_i}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2 dx_i, \quad i = 1, 2, \ldots, n,$$

$$U_I(x_I,t) = -\frac{S_I(t)}{(EA)_I}x_I - \frac{1}{2}\int_0^{x_I}\left(\frac{\partial W_I(x_I,t)}{\partial x_I}\right)^2 dx_I.$$

Boundary conditions:

$$W_I(0,t) = W_1(0,t) = 0,$$

$$U_I(0,t) = U_1(0,t) = 0,$$

$$\left.\frac{\partial W_I(x_I,t)}{\partial x_I}\right|_{x_I=0} = \left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|_{x_1=0} = 0,$$

$$W_I(L,t) = W_n(l,t),$$

$$U_I(L,t) = U_n(l,t),$$

$$\left.\frac{\partial W_I(x_I,t)}{\partial x_I}\right|_{x_I=L} = \left.\frac{\partial W_n(x_n,t)}{\partial x_n}\right|_{x_n=l},$$

$$W_j(l,t) = W_{j+1}(0,t),$$

$$U_j(l,t) = U_{j+1}(0,t),$$

$$(EJ)_I\left.\frac{\partial^2 W_I(x_I,t)}{\partial x_I^2}\right|_{x_I=L} + (EJ)_n\left.\frac{\partial^2 W_n(x_n,t)}{\partial x_n^2}\right|_{x_n=l} + P\left[\Gamma\left.\frac{\partial W_n(x_n,t)}{\partial x_n}\right|_{x_n=l} + \Theta W_n(l,t)\right] = 0,$$

$$(EJ)_I\left.\frac{\partial^3 W_I(x_I,t)}{\partial x_I^3}\right|_{x_I=L} + (EJ)_n\left.\frac{\partial^3 W_n(x_n,t)}{\partial x_n^3}\right|_{x_n=l} + P\left[\Lambda\left.\frac{\partial W_n(x_n,t)}{\partial x_n}\right|_{x_n=l} + \Xi W_n(l,t)\right] - m\frac{\partial^2 W_n(l,t)}{\partial t^2} = 0,$$

$$\left.\frac{\partial W_j(x_j,t)}{\partial x_j}\right|_{x_j=l} = \left.\frac{\partial W_{j+1}(x_{j+1},t)}{\partial x_{j+1}}\right|_{x_{j+1}=0}, \quad j = 1, 2, \ldots, n-1.$$

	Γ	Θ	Λ	Ξ
	$\frac{(r-l_0)(R-l_0)}{R-r}$	$-\frac{r-l_0}{R-r}$	$\frac{R-l_0}{R-r}$	$-\frac{1}{R-r}$
$r = l_0$	0	0	1	$-\frac{1}{R-r}$
$r = R = l_0$	0	0	1	∞
$r = R$	$W_n(l,t) = (R-l_0)\left.\frac{\partial W_n(x_n,t)}{\partial x_n}\right|_{x_n=l}$; $\left.\frac{\partial^3 W_n(x_n,t)}{\partial x_n^3}\right|_{x_n=l} - \frac{1}{(R-l_0)}\left.\frac{\partial^2 W_n(x_n,t)}{\partial x_n^2}\right|_{x_n=l} + -\frac{m}{(EJ)_n}\frac{\partial^2 W_n(l,t)}{\partial t^2} = 0$			

RESULTS

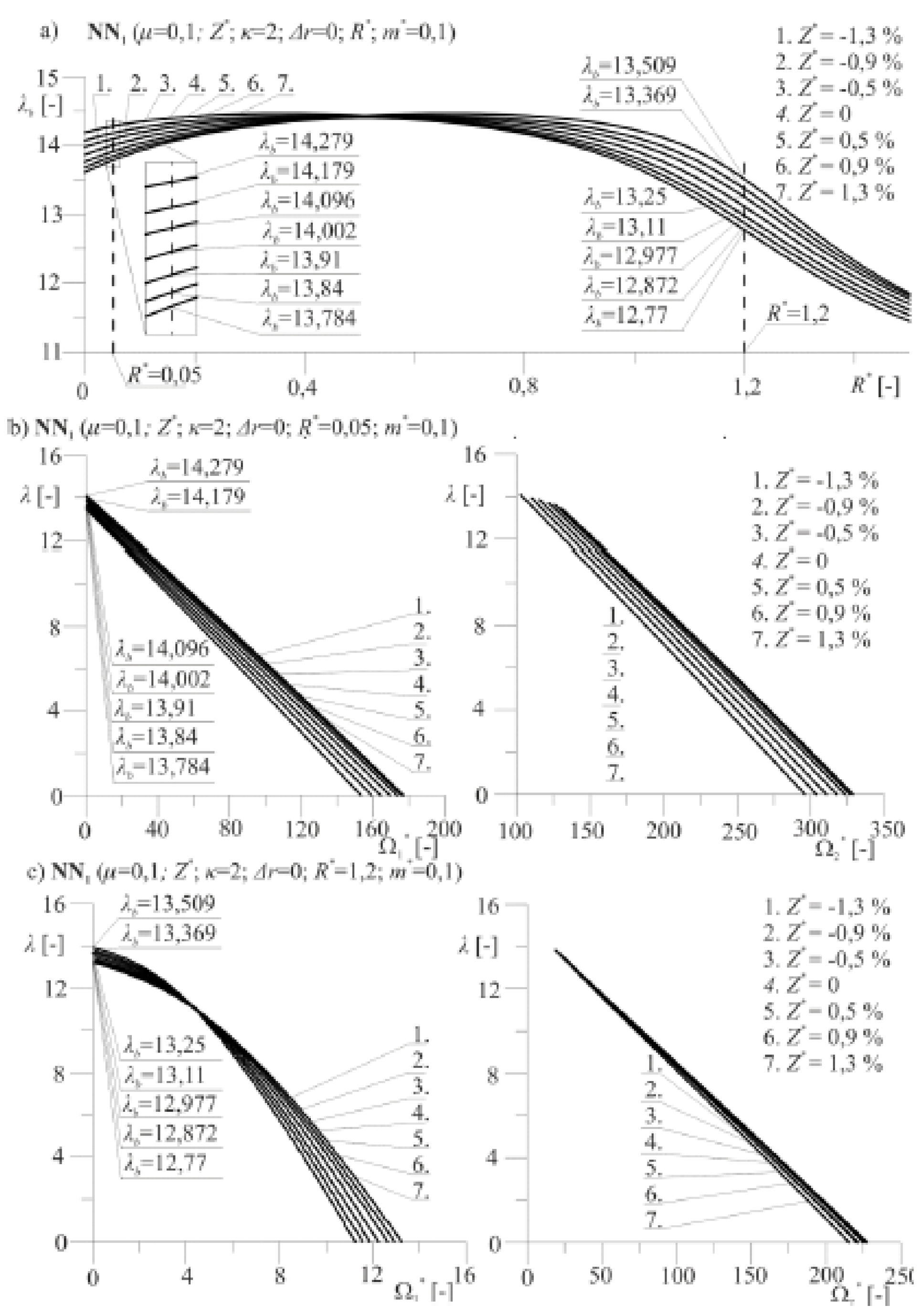

Fig. 3. Eigenvalues of column NN_1: a) bifurcation load in function of loading head radius; vibration frequences for selected values of Z^* and b) $R^* = 0.05$, c) $R^*=1.2$

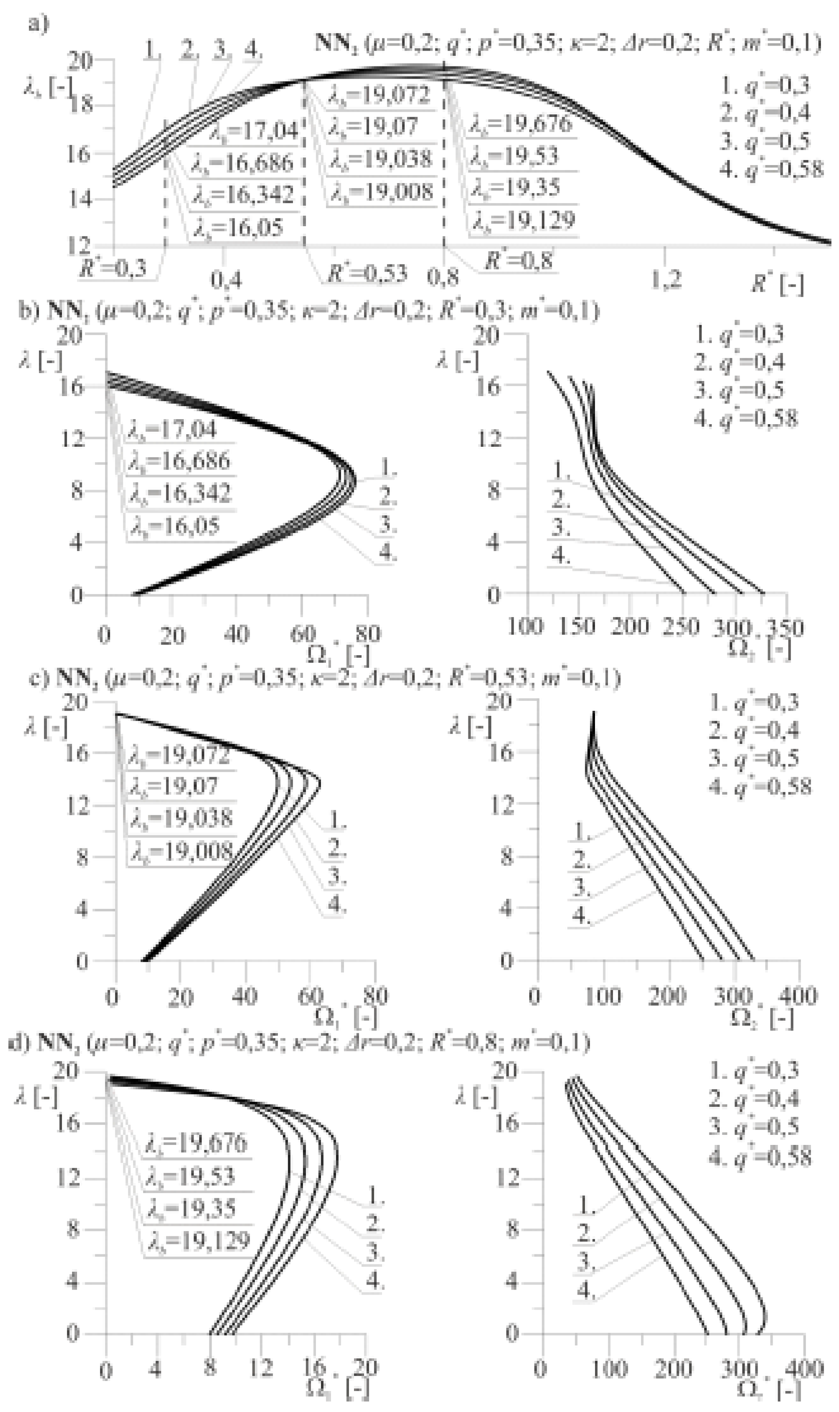

Fig. 4. Eigenvalues of column NN_2: a) bifurcation load in function of loading head radius; vibration frequences for selected values of Z^* and b) $R^* = 0.3$, c) $R^*=0.53$, d) $R^*=0.8$

Dimensionless parameters:

$$\lambda_b = \frac{P_b L^2}{(EJ)_{pr}} \qquad \Omega_i^* = \frac{(\rho A)_{pr}\,\omega_i^2 L^4}{(EJ)_{pr}}$$

$$r^* = \frac{r - l_0}{L} \qquad \Delta r = \frac{R - r}{L}$$

$$Z^* = \frac{b_1 - b_n}{L} \qquad p^* = \frac{p}{L} \qquad q^* = \frac{q}{b_{pr}}$$

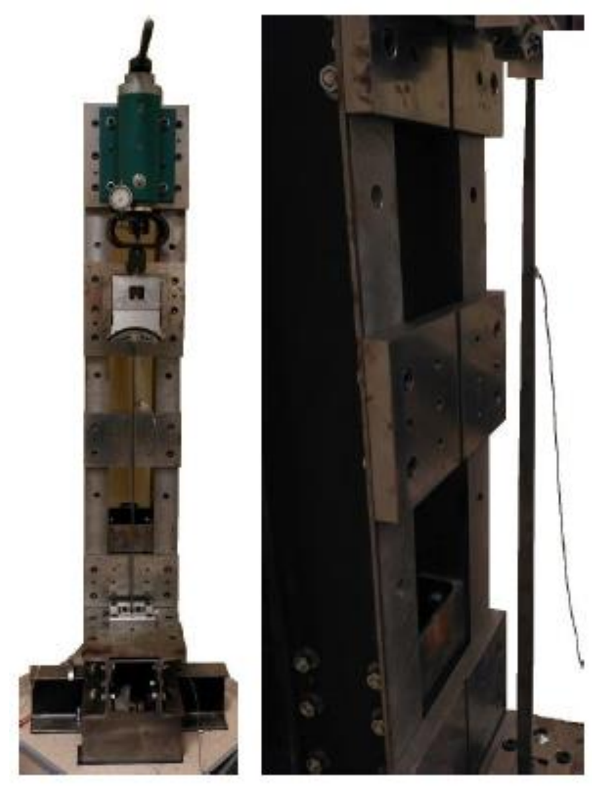

Fig. 5. Research stand for frequency studies of slender rod systems

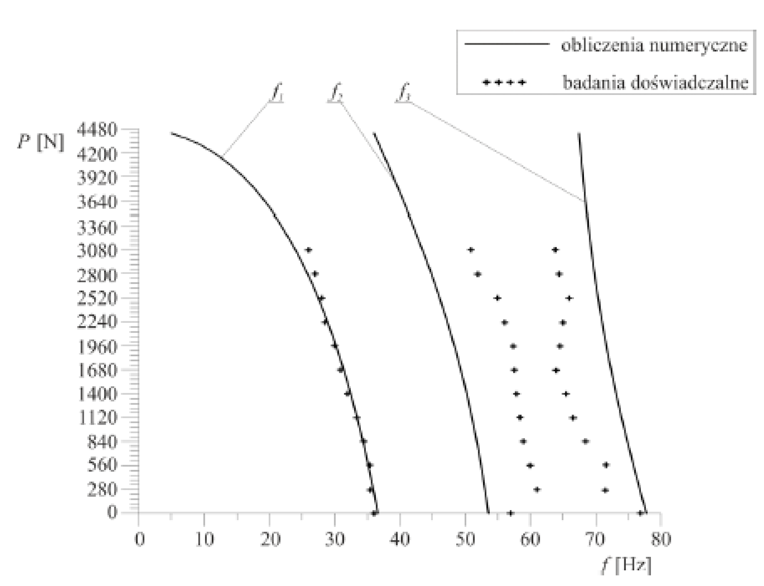

Fig. 6. Frequences in relations to external load – comparison of theoretical and experimental results – NN_2 (R^*=0.10125, μ=0.22, p^*=0.75, q^*=0.6, m^*=1.477)

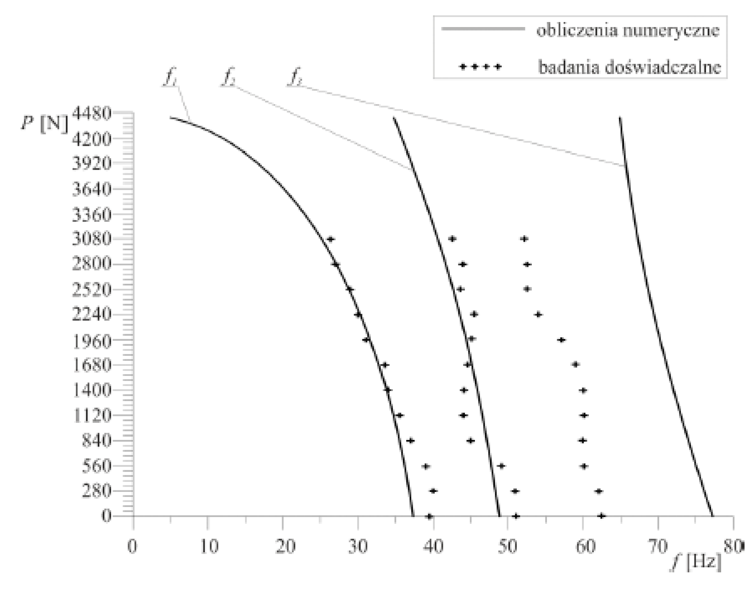

Fig. 7. Frequences in relations to external load – comparison of theoretical and experimental results – NN_1 (R^*=0.10125, μ=0.22, p^*=0.25, q^*=0.6, m^*=1.477)

REMARKS

The work includes theoretical considerations, numerical analysis and experimental research on the issue of free vibrations of a geometrically nonlinear column within the range of a rectilinear form of static equilibrium. The column was loaded with a follower force directed towards the positive pole a case of specific load. The problem was formulated on the basis of Bernoulli - Euler's theory and Hamilton's principle. Due to the occurring geometric nonlinearity, in order to obtain a solution, the perturbative method was used - the small parameter of amplitude method.

The research results illustrate the influence of the variable shape of the rod as a component of a complex slender column on the values of the bifurcation load and natural frequency. On the basis of a series of tests, the most advantageous rod shape outlines were determined in terms of column strength. The possibility of controlling the dynamic properties of the system by means of variable parameters of the approximating functions was also analyzed. The research was experimentally verified in terms of the variability of the natural frequency as a function of the external load. Potential reasons for the differences between theoretical and experimental research were indicated.